

# Light stringy states

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**Pascal Anastasopoulos<sup>1,\*</sup>, Massimo Bianchi<sup>2,†</sup>, Robert Richter<sup>2,3,‡</sup>**

<sup>1</sup> *Technische Univ. Wien Inst. für Theoretische Physik, A-1040 Vienna, Austria*

<sup>2</sup> *Dipartimento di Fisica & Sezione I.N.F.N., Università di Roma “Tor Vergata”,  
Via della Ricerca Scientifica, 00133 Roma, Italy*

<sup>3</sup> *II. Institut für Theoretische Physik, Hamburg University, Germany*

**ABSTRACT:** We carefully study the spectrum of open strings localized at the intersections of D6-branes and identify the lowest massive ‘twisted’ states and their vertex operators, paying particular attention to the signs of the intersection angles. We argue that the masses of the lightest states scale as  $M_\theta^2 \approx \theta M_s^2$  and can thus be parametrically smaller than the string scale. Relying on previous analyses, we compute scattering amplitudes of massless ‘twisted’ open strings and study their factorization, confirming the presence of the light massive states as sub-dominant poles in one of the channels.

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\*pascal@hep.itp.tuwien.ac.at

†Massimo.Bianchi@roma2.infn.it

‡robert.richter@desy.de

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## 1. Introduction

Vacuum configurations with open unoriented strings have attracted a lot of attention in the past few years for their remarkable phenomenological properties [1–4]<sup>1</sup>. One of the peculiar features is the possibility of accommodating large extra dimensions giving rise to a significantly lower string scale, even of a few  $TeV$  [8–10]. Scenarios of these kinds may explain the hierarchy problem, but also allow for stringy signatures that can be observed at LHC [11–24].

Recently, in a series of papers [25–28] the authors study tree-level string scattering amplitudes containing at most two chiral fermions. They show that these amplitudes exhibit a universal behaviour independently of the specifics of the compactification, which gives their results a predictive power. The observed poles correspond to the exchanges of Regge excitations of the standard model gauge bosons,

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<sup>1</sup>For reviews on phenomenological implications of D-instantons in this context, see [5–7]

whose masses scale with the string mass  $M_s$ . On the other hand there exist a tower of stringy excitations of the chiral fermions and their superpartners localized at the intersections of two stacks of D-branes. Their masses depend on the string mass  $M_s$  and the intersection angle  $\theta$  and thus can be significantly lighter than the Regge excitations of the gauge bosons.

A large subclass of semi-realistic global D-brane constructions exhibit small intersection angles between two stacks of D-branes and thus allow for light stringy states. *A priori* the widths of the angles depend on the wrapping numbers of the intersecting branes and on the moduli of the compactification, associated to closed-string excitations. Playing with both discrete and continuous degrees of freedom it is possible to lower the threshold for the production of these states well below the string scale  $M_s \approx \sqrt{T_s}$ . Similar considerations apply to (generalized) Kaluza-Klein excitations, that we will not delve upon very much here. Aim of the present work is the investigation of massive, but potentially very light, open string states. We analyze in detail a configuration of intersecting D-branes, discuss the states arising at such an intersection beyond the massless level. Moreover, we give a detailed description for the construction of their vertex operators, which crucially depends on the signs of the intersection angles.

Equipped with the vertex operators for arbitrary intersection angles we compute the four point amplitude containing four fermions. We investigate various limits of this amplitude and show that the most dominant poles correspond to the exchanges of the light stringy states. While the signals of such light stringy states at colliders could be not so easy to recognize and discriminate from other kinds of Physics Beyond the Standard Model the amplitude also exhibits signatures of higher spin exchanges, whose origin is purely stringy and whose masses do not vanish for small angles. Thus signatures of light stringy states may provide a first step towards evidence for string theory.

The presentation will be organized as follows. In section 2, we discuss a local configuration of two intersecting D-brane stacks, analyze the states localized at such intersection and eventually display their corresponding masses and vertex operators. In section 3 we will compute some relevant scattering amplitudes at tree-level (disk) and expose the massive poles associated to massive, but light open strings. In section 4 we will conclude. The appendices A and B provide some technical details necessary for the analysis.

## 2. Quantization of strings localized at D-brane intersections

In this section we will analyze the states localized at the intersection of two stacks of D6-branes. We will investigate under which conditions the states are massless and

identify their corresponding vertex operators<sup>2</sup>. An open string stretched between two D-brane stacks intersecting at an angle  $\pi\theta$  in the  $(X, Y)$  plane has to fulfil the boundary conditions [33–35]

$$\begin{aligned}\partial_\sigma X(\tau, 0) &= 0 = Y(\tau, 0) \\ \partial_\sigma X(\tau, \pi) + \tan(\pi\theta) \partial_\sigma Y(\tau, \pi) &= 0 \\ Y(\tau, \pi) - \tan(\pi\theta) X(\tau, \pi) &= 0.\end{aligned}\tag{2.1}$$

It proves convenient to introduce complex coordinates  $Z^I = X^I + iY^I$  with  $I = 1, 2, 3$  for the internal (compactified) directions. Given these boundary conditions for the  $X^I, Y^I$ , one can deduce the mode expansions for  $Z^I$  that read

$$\left. \begin{aligned} Z^I(z, \bar{z}) &= -\sum_n \frac{\alpha_{n-\theta_I}^I}{(n-\theta_I)} z^{-n+\theta_I} - \sum_n \frac{\alpha_{n+\theta_I}^I}{(n+\theta_I)} \bar{z}^{-n-\theta_I} \\ \bar{Z}^I(z, \bar{z}) &= -\sum_n \frac{\alpha_{n+\theta_I}^I}{(n+\theta_I)} z^{-n-\theta_I} - \sum_n \frac{\alpha_{n-\theta_I}^I}{(n-\theta_I)} \bar{z}^{-n+\theta_I} \end{aligned} \right\} \text{for } I = 1, 2, 3. \tag{2.2}$$

After applying the doubling trick in order to extend the upper plane to the whole complex plane one gets for the conformal fields  $\partial Z$  and  $\partial \bar{Z}$

$$\partial Z(z) = \sum_n \alpha_{n-\theta_I}^I z^{-n+\theta_I-1} \quad \partial \bar{Z}(z) = \sum_n \alpha_{n+\theta_I}^I z^{-n-\theta_I-1}. \tag{2.3}$$

Upon quantization the only non-vanishing commutators are

$$[\alpha_{n\pm\theta}^I, \alpha_{m\mp\theta}^{I'}] = (m \pm \theta) \delta_{n+m} \delta^{II'}.$$

World-sheet supersymmetry

$$\delta X^i = \bar{\epsilon} \psi^i$$

leads to the same moding for the complexified world-sheet fermions. For the NS sector one obtains (again after using the doubling trick)

$$\Psi^I(z) = \sum_{r \in \mathbb{Z} + \frac{1}{2}} \psi_{r-\theta_I}^I z^{-r-\frac{1}{2}+\theta_I} \quad \bar{\Psi}^I(z) = \sum_{r \in \mathbb{Z} + \frac{1}{2}} \psi_{r+\theta_I}^I \bar{z}^{-r-\frac{1}{2}-\theta_I}, \tag{2.4}$$

Upon quantization the only non-vanishing anti-commutator are

$$\{\psi_{m-\theta_I}^I, \psi_{n+\theta_I}^{I'}\} = \delta_{m,n} \delta^{II'}. \tag{2.5}$$

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<sup>2</sup>For a detailed discussion of vertex operators for massless states at arbitrary intersection angles, see [29, 30]. For a discussion of instantonic modes at the intersection of D-instanton and D-brane at arbitrary angles see [31]. For a discussion of vertex operators for massive states in heterotic compactifications, see [32].

On the other hand for the R-sector one gets

$$\Psi^I(z) = \sum_{r \in \mathbb{Z}} \psi_{r-\theta_I}^I z^{-r-\frac{1}{2}+\theta_I} \quad \bar{\Psi}^I(z) = \sum_{r \in \mathbb{Z}} \psi_{r+\theta_I}^I \bar{z}^{-r-\frac{1}{2}-\theta_I} , \quad (2.6)$$

where in contrast to the NS-sector the mode expansion is over integers instead of half-integers. Upon quantization the only non-vanishing anti-commutators are again (2.5) where  $m$  and  $n$  are now integers.

In the following we will analyze the spectrum. To this end we need to properly define the ground-state and identify the annihilation and creation operators. Moreover, we will determine the mass of various states and finally we derive their corresponding vertex operator. We start with the NS-sector which describes space-time bosons and then turn to the R-sector which describes space-time fermions.

## 2.1 NS-sector

The definition of the ground-state crucially depends on whether the intersection angles are positive or negative. Thus we distinguish between two different scenarios and investigate them separately.

### Positive intersection angle

By restricting our attention onto just one complex dimension for the moment, we start with analyzing the setup where the intersection angle is positive. For a positive intersection angle the ground-state  $|\theta_I\rangle$  is defined as

$$\begin{aligned} \alpha_{m-\theta_I}|\theta_I\rangle &= 0 & m \geq 1 & & \psi_{r-\theta_I}|\theta_I\rangle &= 0 & r \geq \frac{1}{2} \\ \alpha_{m+\theta_I}|\theta_I\rangle &= 0 & m \geq 0 & & \psi_{r+\theta_I}|\theta_I\rangle &= 0 & r \geq \frac{1}{2} . \end{aligned} \quad (2.7)$$

Later in subsection 2.1.1, where we investigate states localized at the intersection of two D6-branes, we will see that the mass of the states is determined by the Virasoro generator  $L_0 = \sum_{I=0}^3 L_0^I$ , where  $L_0^I$  denotes the Virasoro generator of the  $I$ -th complex dimension and reads

$$L_0^I = \sum_{m \in \mathbb{Z}} : \alpha_{-m+\theta_I}^I \alpha_{m-\theta_I}^I : + \sum_{m \in \mathbb{Z}} (m - \theta_I) : \psi_{-m+\theta_I}^I \psi_{m-\theta_I}^I : + \epsilon_0^I . \quad (2.8)$$

Here  $\epsilon_0^I$  denotes the zero point energy of the  $I$ -th dimension that can be computed by  $\zeta$ -function regularization, as we will demonstrate momentarily

$$\begin{aligned} \epsilon_0^I &= \sum_{m=-\infty}^0 [\alpha_{-m+\theta_I}^I, \alpha_{m-\theta_I}^I] + \sum_{m=-\infty}^{-1/2} (r - \theta_I) \{\psi_{-m+\theta_I}^I, \psi_{m-\theta_I}^I\} \\ &= \zeta[-1, \theta_I] - \zeta[-1, 1/2 + \theta_I] = -\frac{1}{8} + \frac{1}{2} \theta_I . \end{aligned} \quad (2.9)$$

Due to the non-trivial intersection angles the vertex operators describing the states under consideration involve bosonic and fermionic twist fields accounting for the boundary conditions (2.1). In order to properly identify these twist fields we determine the action of the conformal fields  $\Psi^I$ ,  $\bar{\Psi}^I$ ,  $\partial Z^I$  and  $\partial \bar{Z}^I$  on the ground-state  $|\theta_I\rangle$  and excitations (fermionic and bosonic ones) thereof.

We start with the fermionic conformal fields  $\Psi^I$  and  $\bar{\Psi}^I$ . Let us first examine the ground-state  $|\theta_I\rangle$  whose fermionic part can be identified with the spin field  $s_{\theta_I}^+(0)|0\rangle_u$

$$\begin{aligned}\Psi^I(z)|\theta_I\rangle &= \sum_{r=-\infty}^{\infty} z^{-r-\frac{1}{2}+\theta_I} \psi_{r-\theta_I}|\theta_I\rangle \\ &= \sum_{r=-\infty}^{-\frac{1}{2}} z^{-r-\frac{1}{2}+\theta_I} \psi_{r-\theta_I}|\theta_I\rangle \sim z^{\theta_I} \psi_{-\frac{1}{2}-\theta_I}|\theta_I\rangle = z^{\theta_I} \tilde{t}_{\theta_I}^+(0)|0\rangle_u.\end{aligned}$$

Here  $|0\rangle_u$  denotes the untwisted ground-state, while  $\tilde{t}_{\theta_I}^+$  denotes the excited twist field whose conformal dimension is  $h_{\tilde{t}_{\theta_I}^+} = \frac{1}{2}(1+\theta_I)^2$ . Similarly we obtain for  $\bar{\Psi}^I(z)|\theta_I\rangle$

$$\bar{\Psi}^I(z)|\theta_I\rangle \rightarrow z^{-\theta_I} \psi_{-\frac{1}{2}+\theta_I}|\theta_I\rangle = z^{-\theta_I} t_{\theta_I}^+(0)|0\rangle_u,$$

where  $t_{\theta_I}^+$  is another excited twist field with conformal dimension  $h_{t_{\theta_I}^+} = \frac{1}{2}(1-\theta_I)^2$ . Using the same procedure, the actions of  $\Psi^I$  and  $\bar{\Psi}_I$  on the state  $\psi_{-\frac{1}{2}+\theta_I}|\theta_I\rangle$  read

$$\begin{aligned}\Psi^I(z)\psi_{-\frac{1}{2}+\theta_I}|\theta_I\rangle &\rightarrow z^{\theta_I-1}|\theta_I\rangle = z^{\theta_I-1} s_{\theta_I}^+(0)|0\rangle_u \\ \bar{\Psi}^I(z)\psi_{-\frac{1}{2}+\theta_I}|\theta_I\rangle &\rightarrow z^{1-\theta_I} \psi_{-\frac{3}{2}+\theta_I} \psi_{-\frac{1}{2}+\theta_I}|\theta_I\rangle = z^{1-\theta_I} u_{\theta_I}^+(0)|0\rangle_u.\end{aligned}$$

Here  $u_{\theta_I}^+$  is a doubly excited twist field with conformal dimension  $h_{u_{\theta_I}^+} = \frac{1}{2}(2-\theta_I)^2$ . For the fermionic sector these considerations will be sufficient for anything we will do later (since higher excitations will be necessarily massive even in the limit  $\theta_I \rightarrow 0$ ), but in principle this procedure can be applied to any other more complicated state. The fermionic conformal fields can be bosonized leading to

$$\begin{aligned}\Psi^I &= e^{iH_I} & h_{\Psi} &= \frac{1}{2} \\ \bar{\Psi}^I &= e^{-iH_I} & h_{\bar{\Psi}} &= \frac{1}{2} \\ s_{\alpha}^+ &= e^{i\alpha H_I} & h_{s_{\alpha}^+} &= \frac{1}{2}\alpha^2 \\ \tilde{t}_{\alpha}^+ &= e^{i(1+\alpha)H_I} & h_{\tilde{t}_{\alpha}^+} &= \frac{1}{2}(1+\alpha)^2 \\ t_{\alpha}^+ &= e^{-i(1-\alpha)H_I} & h_{t_{\alpha}^+} &= \frac{1}{2}(1-\alpha)^2 \\ u_{\alpha}^+ &= e^{-i(2-\alpha)H_I} & h_{u_{\alpha}^+} &= \frac{1}{2}(2-\alpha)^2.\end{aligned}\tag{2.10}$$

On the right hand side we display their corresponding conformal dimension.

Let us turn to bosonic sector and apply the same procedure to the bosonic conformal fields  $\partial Z^I$  and  $\partial \bar{Z}^I$ . Again we start with the ground-state  $|\theta_I\rangle$  whose bosonic part will be identified as  $\sigma_{\theta_I}^+(0)|0\rangle_u$

$$\begin{aligned}\partial Z^I(z)|\theta_I\rangle &= \sum_{n=-\infty}^{\infty} \alpha_{n-\theta_I}^I z^{-n+\theta_I-1} |\theta_I\rangle = \sum_{n=-\infty}^0 \alpha_{n-\theta_I}^I z^{-n+\theta_I-1} |\theta_I\rangle \\ &\rightarrow z^{\theta_I-1} \alpha_{-\theta_I}^I |\theta_I\rangle = z^{\theta_I-1} \tau_{\theta_I}^+(0)|0\rangle_u\end{aligned}\quad (2.11)$$

$$\begin{aligned}\partial \bar{Z}^I(z)|\theta_I\rangle &= \sum_{n=-\infty}^{\infty} \alpha_{n+\theta_I}^I z^{-n-\theta_I-1} |\theta_I\rangle = \sum_{n=-\infty}^{-1} \alpha_{n+\theta_I}^I z^{-n-\theta_I-1} |0\rangle \\ &\rightarrow z^{-\theta_I} \alpha_{-1+\theta_I}^I |\theta_I\rangle = z^{-\theta_I} \tilde{\tau}_{\theta_I}^+(0)|0\rangle_u.\end{aligned}\quad (2.12)$$

Here  $\tau_{\theta_I}^+$  and  $\tilde{\tau}_{\theta_I}^+$  denote excited twist fields with conformal dimensions  $h_{\tau_{\theta_I}^+} = \frac{1}{2}\theta_I(3 - \theta_I)$  and  $h_{\tilde{\tau}_{\theta_I}^+} = \frac{1}{2}(1 - \theta_I)(2 + \theta_I)$ , respectively.

For later purposes we also need the action on the excited states  $\alpha_{-\theta_I}^I |\theta_I\rangle$  as well as  $(\alpha_{-\theta_I}^I)^2 |\theta_I\rangle$ . We start with  $\alpha_{-\theta_I}^I |\theta_I\rangle$  which we identified above with  $\tau_{\theta_I}^+ |0\rangle_u$

$$\begin{aligned}\partial Z^I(z) \alpha_{-\theta_I}^I |\theta_I\rangle &= \sum_{n=-\infty}^{\infty} \alpha_{n-\theta_I}^I z^{-n+\theta_I-1} \alpha_{-\theta_I}^I |\theta_I\rangle \\ &= \sum_{n=-\infty}^0 \alpha_{n-\theta_I}^I z^{-n+\theta_I-1} \alpha_{-\theta_I}^I |\theta_I\rangle \\ &\rightarrow z^{\theta_I-1} (\alpha_{-\theta_I}^I)^2 |\theta_I\rangle = z^{\theta_I-1} \omega_{\theta_I}^+(0)|0\rangle_u\end{aligned}\quad (2.13)$$

$$\begin{aligned}\partial \bar{Z}^I(z) \alpha_{-\theta_I}^I |\theta_I\rangle &= \sum_{n=-\infty}^{\infty} \alpha_{n+\theta_I}^I z^{-n-\theta_I-1} \alpha_{-\theta_I}^I |\theta_I\rangle \\ &= \sum_{n=-\infty}^0 \alpha_{n+\theta_I}^I z^{-n-\theta_I-1} \alpha_{-\theta_I}^I |\theta_I\rangle \\ &\rightarrow z^{-1-\theta_I} |\theta_I\rangle = z^{-1-\theta_I} \sigma_{\theta_I}^+(0)|0\rangle_u.\end{aligned}\quad (2.14)$$

The field  $\omega_{\theta_I}^+$  is a doubly excited bosonic twist field with conformal dimension  $h_{\omega_{\theta_I}^+} = \frac{1}{2}\theta_I(5 - \theta_I)$ .

Finally, we turn to  $(\alpha_{-\theta_I}^I)^2 |\theta_I\rangle$  which is identified with  $\omega_\theta^+(0) |\theta_I\rangle_u$

$$\begin{aligned} \partial Z^I(z) (\alpha_{-\theta_I}^I)^2 |\theta_I\rangle &= \sum_{n=-\infty}^{\infty} \alpha_{n-\theta_I}^I z^{-n+\theta_I-1} (\alpha_{-\theta_I}^I)^2 |\theta_I\rangle \\ &= \sum_{n=-\infty}^0 \alpha_{n-\theta_I}^I z^{-n+\theta_I-1} (\alpha_{-\theta_I}^I)^2 |\theta_I\rangle \\ &\rightarrow z^{\theta_I-1} (\alpha_{-\theta_I}^I)^3 |\theta_I\rangle = z^{\theta_I-1} \rho_{\theta_I}^+(0) |0\rangle_u \end{aligned} \quad (2.15)$$

$$\begin{aligned} \partial \bar{Z}^I(z) (\alpha_{-\theta_I}^I)^2 |\theta_I\rangle &= \sum_{n=-\infty}^{\infty} \alpha_{n+\theta_I}^I z^{-n-\theta_I-1} (\alpha_{-\theta_I}^I)^2 |\theta_I\rangle \\ &= \sum_{n=-\infty}^0 \alpha_{n+\theta_I}^I z^{-n-\theta_I-1} (\alpha_{-\theta_I}^I)^2 |\theta_I\rangle \\ &\rightarrow z^{-1-\theta_I} \alpha_{-\theta_I}^I |\theta_I\rangle = z^{-1-\theta_I} \tau_{\theta_I}^+(0) |0\rangle_u . \end{aligned} \quad (2.16)$$

Here  $\rho_{\theta_I}^+$  denotes the triply excited bosonic twist field with conformal dimension  $h_{\rho_{\theta_I}^+} = \frac{1}{2}\theta_I(7 - \theta_I)$ .

Below we summarize our findings and display the various bosonic twist fields and their corresponding conformal dimensions

$$\begin{aligned} \sigma_\alpha^+ : \quad & h_{\sigma_\alpha^+} = \frac{1}{2}\alpha(1 - \alpha) \\ \tau_\alpha^+ : \quad & h_{\tau_\alpha^+} = \frac{1}{2}\alpha(3 - \alpha) \\ \tilde{\tau}_\alpha^+ : \quad & h_{\tilde{\tau}_\alpha^+} = \frac{1}{2}(1 - \alpha)(2 + \alpha) \\ \omega_\alpha^+ : \quad & h_{\omega_\alpha^+} = \frac{1}{2}\alpha(5 - \alpha) \\ \rho_\alpha^+ : \quad & h_{\rho_\alpha^+} = \frac{1}{2}\alpha(7 - \alpha) . \end{aligned} \quad (2.17)$$

### Negative intersection angle

Here we investigate the scenario of a negative intersection angle. For such a setup the ground-state  $|\theta_I\rangle$  is defined differently, namely it satisfies

$$\begin{aligned} \alpha_{m-\theta_I}^I |\theta_I\rangle &= 0 \quad m \geq 0 & \psi_{r-\theta_I}^I |\theta_I\rangle &= 0 \quad r \geq \frac{1}{2} \\ \alpha_{m+\theta_I}^I |\theta_I\rangle &= 0 \quad m \geq 1 & \psi_{r+\theta_I}^I |\theta_I\rangle &= 0 \quad r \geq \frac{1}{2} \end{aligned} \quad (2.18)$$

The Virasoro generator  $L_0^I$  takes basically the same form as before for the setup with a positive intersection angle

$$L_0^I = \sum_{m \in \mathbb{Z}} : \alpha_{-m+\theta_I}^I \alpha_{m-\theta_I}^I : + \sum_{m \in \mathbb{Z}} (m - \theta_I) : \psi_{-m+\theta_I}^I \psi_{m-\theta_I}^I : + \epsilon_0^I , \quad (2.19)$$



where the zero point energy computes to (recall the angle  $\theta_I$  is negative)

$$\epsilon_0^I = -\frac{1}{8} - \frac{1}{2}\theta_I . \quad (2.20)$$

Again we examine the action of the fermionic fields  $\Psi(z)$  and  $\bar{\Psi}(z)$  on the ground-state  $|\theta_I\rangle$  as well as on some excited states. For  $|\theta_I\rangle$  we get

$$\begin{aligned} \Psi^I(z) |\theta_I\rangle &\longrightarrow z^{\theta_I} \psi_{-\frac{1}{2}-\theta_I} |\theta_I\rangle = z^{\theta_I} t_{-\theta_I}^-(0) |0\rangle_u \\ \bar{\Psi}^I(z) |\theta_I\rangle &\longrightarrow z^{-\theta_I} \psi_{-\frac{1}{2}+\theta_I} |\theta_I\rangle = z^{-\theta_I} \tilde{t}_{-\theta_I}^-(0) |0\rangle_u \end{aligned}$$

and for  $\psi_{-\frac{1}{2}-\theta_I} |\theta_I\rangle$

$$\begin{aligned} \Psi^I(z) \psi_{-\frac{1}{2}-\theta_I} |\theta_I\rangle &\longrightarrow z^{1+\theta_I} \psi_{-\frac{3}{2}-\theta_I} |\theta_I\rangle = z^{1+\theta_I} u_{-\theta_I}^-(0) |0\rangle_u \\ \bar{\Psi}^I(z) \psi_{-\frac{1}{2}-\theta_I} |\theta_I\rangle &\longrightarrow z^{-1-\theta_I} |\theta_I\rangle = z^{-1-\theta_I} s_{-\theta_I}^-(0) |0\rangle_u . \end{aligned}$$

Upon bosonization the twist fields take the form (recall the intersection angle  $\theta_I$  is negative)

$$\begin{aligned} s_\alpha^- &= e^{-i\alpha H_I} & h_{s_\alpha^-} &= \frac{1}{2}\alpha^2 \\ t_\alpha^- &= e^{i(1-\alpha)H_I} & h_{t_\alpha^-} &= \frac{1}{2}(1-\alpha)^2 \\ \tilde{t}_\alpha^- &= e^{-i(1+\alpha)H_I} & h_{\tilde{t}_\alpha^-} &= \frac{1}{2}(1+\alpha)^2 \\ u_\alpha^- &= e^{i(2-\alpha)H_I} & h_{u_\alpha^-} &= \frac{1}{2}(2-\alpha)^2 , \end{aligned} \quad (2.21)$$

where, again, we display the conformal dimension of the respective twist fields.

Following the same procedure for the bosonic sector gives the action of the conformal fields  $\partial Z^I, \partial \bar{Z}^I$  on the ground-state  $|\theta_I\rangle$  that corresponds to the bosonic anti-twist field  $\sigma_{-\theta_I}^-(0) |0\rangle_u$

$$\begin{aligned} \partial Z^I(z) |\theta_I\rangle &\longrightarrow z^{\theta_I} \alpha_{-1-\theta_I} |\theta_I\rangle = z^{\theta_I} \tau_{-\theta_I}^-(0) |0\rangle_u \\ \partial \bar{Z}^I(z) |\theta_I\rangle &\longrightarrow z^{-1-\theta_I} \alpha_{\theta_I} |\theta_I\rangle = z^{-1-\theta_I} \tilde{\tau}_{-\theta_I}^-(0) |0\rangle_u . \end{aligned}$$

Here  $\tau_{-\theta_I}^-$  and  $\tilde{\tau}_{-\theta_I}^-$  are excited bosonic anti-twist fields with the conformal dimension  $h_{\tilde{\tau}_{-\theta_I}^-} = \frac{1}{2}(1+\theta_I)(2-\theta_I)$  and  $h_{\tau_{-\theta_I}^-} = -\frac{1}{2}\theta_I(3+\theta_I)$ .

As before for the positive intersection angle let us perform the same analysis also for bosonic excitations of the ground-state. We start with  $\alpha_{\theta_I} |\theta_I\rangle$ , which corresponds to the excited twist field  $\tau_\theta^-(0) |0\rangle_u$ ,

$$\begin{aligned} \partial Z^I(z) \alpha_{\theta_I} |\theta_I\rangle &\longrightarrow z^{\theta_I-1} |\theta_I\rangle = z^{\theta_I-1} \sigma_{\theta_I}^-(0) |0\rangle_u \\ \partial \bar{Z}^I(z) \alpha_{\theta_I} |\theta_I\rangle &\longrightarrow z^{-1-\theta_I} (\alpha_{\theta_I})^2 |\theta_I\rangle = z^{-1-\theta_I} \omega_{\theta_I}^-(0) |0\rangle_u . \end{aligned}$$

Finally, we also look at the state  $(\alpha_{\theta_I})^2 |\theta_I\rangle$ , which corresponds to the excited twist field  $\tilde{\omega}_{\theta_I}^-(z)$ . One obtains

$$\begin{aligned} \partial Z^I(z) (\alpha_{\theta_I})^2 |\theta_I\rangle &\longrightarrow z^{\theta_I-1} \alpha_{\theta_I} |\theta_I\rangle = z^{\theta_I-1} \tau_{\theta_I}^-(0) |0\rangle_u \\ \partial \bar{Z}^I(z) (\alpha_{\theta_I})^2 |\theta_I\rangle &\longrightarrow z^{-1-\theta_I} (\alpha_{\theta_I})^3 |\theta_I\rangle = z^{-1-\theta_I} \rho_{\theta_I}^-(0) |0\rangle_u . \end{aligned}$$

Below, we display the conformal dimensions of the various twist fields

$$\begin{aligned} \sigma_{\alpha}^- : \quad & h_{\sigma_{\alpha}^-} = -\frac{1}{2}\alpha (1 - \alpha) \\ \tilde{\tau}_{\alpha}^- : \quad & h_{\tilde{\tau}_{\alpha}^-} = \frac{1}{2} (1 - \alpha) (2 + \alpha) \\ \tau_{\alpha}^- : \quad & h_{\tau_{\alpha}^-} = -\frac{1}{2}\alpha (3 - \alpha) \\ \omega_{\alpha}^- : \quad & h_{\omega_{\alpha}^-} = -\frac{1}{2}\alpha (5 - \alpha) \\ \rho_{\alpha}^- : \quad & h_{\rho_{\alpha}^-} = -\frac{1}{2}\alpha (7 - \alpha) . \end{aligned} \tag{2.22}$$

### 2.1.1 States and vertex operators in the NS-sector

Now we have all the ingredients to determine the vertex operators for the lowest excitations of the ‘twisted’ ground-state. Since we are potentially interested in supersymmetric intersections, the three intersection angles have to satisfy the following condition

$$\theta_1 + \theta_2 + \theta_3 = 0 \quad \text{mod } 2 \tag{2.23}$$

which leaves the following four options

- $\theta_1, \theta_2, \theta_3 \geq 0$  with  $\sum_I \theta_I = 2$
- $\theta_1, \theta_2 \geq 0$  and  $\theta_3 \leq 0$  with  $\sum_I \theta_I = 0$
- $\theta_1, \theta_2 \leq 0$  and  $\theta_3 \geq 0$  with  $\sum_I \theta_I = 0$
- $\theta_1, \theta_2, \theta_3 \leq 0$  with  $\sum_I \theta_I = -2$ ,

where the last two options are just the mirror of the first two and are therefore already covered by them. Below we will discuss these two setups in detail, we present the massless states in the NS- and R-sector, display their corresponding vertex operator and then turn to genuinely massive string states discuss their masses as well as their vertex operators.

Since we are interested in the mass of the various states, below we display the mass formula, which can be easily derived from the Virasoro operator  $L_0 = \sum_{I=0}^3 L_0^I$

(note that the mass computation is done in the light-cone gauge) by using the fact that  $\alpha_0^0$  represents the four-dimensional energy (=time component of 4-momentum)

$$M^2 = \left( \sum_{\mu=0}^1 \left\{ \sum_{n \in \mathbb{Z}n} : \alpha_{-n}^\mu \alpha_n^\mu : + \sum_{n \in \mathbb{Z}} n : \psi_{-n}^\mu \psi_n^\mu : \right\} \right. \\ \left. + \sum_{I=1}^3 \left\{ \sum_{m \in \mathbb{Z}} : \alpha_{-m+\theta_I}^I \alpha_{m-\theta_I}^I : + \sum_{m \in \mathbb{Z}} (m - \theta_I) : \psi_{-m+\theta_I}^I \psi_{m-\theta_I}^I : \right\} + \epsilon_0 \right) M_s^2 . \quad (2.24)$$

Here  $\epsilon_0$  denotes again the zero point energy, which crucially depends on the angle.

Finally, not all possible excitations correspond to physical states. The GSO projection, ensuring the modular invariance of the parent closed-string partition function, requires that a physical state in the NS-sector contains an odd number of fermionic excitations.

### Only positive angles

Let us start with the setup in which all intersection angles are positive. In that case the supersymmetry condition reads<sup>3</sup>

$$\theta_1 + \theta_2 + \theta_3 = 2 . \quad (2.25)$$

The lightest states in that case are given by

$$\psi_{-\frac{1}{2}+\theta_1} | \theta_{1,2,3} \rangle \quad M^2 = \frac{1}{2} (-\theta_1 + \theta_2 + \theta_3) M_s^2 \quad (2.26)$$

$$\psi_{-\frac{1}{2}+\theta_2} | \theta_{1,2,3} \rangle \quad M^2 = \frac{1}{2} (\theta_1 - \theta_2 + \theta_3) M_s^2 \quad (2.27)$$

$$\psi_{-\frac{1}{2}+\theta_3} | \theta_{1,2,3} \rangle \quad M^2 = \frac{1}{2} (\theta_1 + \theta_2 - \theta_3) M_s^2 \quad (2.28)$$

$$\prod_{I=1}^3 \psi_{-\frac{1}{2}+\theta_I} | \theta_{1,2,3} \rangle \quad M^2 = \left( 1 - \frac{1}{2} (\theta_1 + \theta_2 + \theta_3) \right) M_s^2 . \quad (2.29)$$

Now we can use what we learned in the analysis above to identify the corresponding vertex operators for the respective states. Here we will make use of the bosonized form of the fermionic twist operator. We start by discussing in detail the state  $\prod_{I=1}^3 \psi_{-\frac{1}{2}+\theta_I} | \theta_{1,2,3} \rangle$  and then display the vertex operators for the other excitations.

Following the rules above the corresponding vertex operator should take the form

$$\prod_{I=1}^3 \psi_{-\frac{1}{2}+\theta_I} | \theta_{1,2,3} \rangle : \quad V_{\phi_4^*}^{(-1)} = \Lambda_{ab} \phi_4^* e^{-\varphi} \prod_{I=1}^3 \sigma_{\theta_I}^+ e^{-i(1-\theta_I)H_I} e^{ikX} . \quad (2.30)$$

It is easy to verify that the conformal dimension of this vertex operator is

$$2 - \frac{1}{2} \sum_{I=1}^3 \theta_I + k^2 \quad (2.31)$$

---

<sup>3</sup>Here all angles lie in the open interval  $(0, 1)$ .

and the state becomes massless once the supersymmetry condition is satisfied. How do we know that one has to identify this state as the lowest component of an anti-chiral superfield rather than of a chiral superfield? This can be answered by looking at the  $U(1)_{WS}$  charge which in the canonical  $(-1)$ -ghost picture is the same as the  $U(1)_R$  charge. In this specific case the  $U(1)_{WS}$  charge is

$$\sum_{I=1}^3 (\theta_I - 1) = -1 \quad \text{for} \quad \theta_1 + \theta_2 + \theta_3 = 2. \quad (2.32)$$

Thus this state should be combined with a right-handed spinor field to form an anti-chiral supermultiplet. The conjugate field is the string going from brane  $b$  to  $a$  and its vertex operator takes the form (keep in mind that the angles from D6-brane  $b$  to D6-brane  $a$  are now  $-\theta_I$  and thus all negative.)

$$V_{\phi_4}^{(-1)} = \Lambda_{ba} \phi_4 e^{-\varphi} \prod_{I=1}^3 \sigma_{\theta_I}^- e^{i(1-\theta_I)H_I} e^{ikX}. \quad (2.33)$$

Below we display the vertex operators for the other three fields

$$\psi_{-\frac{1}{2}+\theta_1} \mid \theta_{1,2,3} \rangle : \quad V_{\phi_1}^{-1} = \Lambda_{ab} \phi_1 e^{-\varphi} \sigma_{\theta_1}^+ e^{-i(1-\theta_1)H_1} \prod_{I \neq 1}^3 \sigma_{\theta_I}^+ e^{i\theta_I H_I} \quad (2.34)$$

$$\psi_{-\frac{1}{2}+\theta_2} \mid \theta_{1,2,3} \rangle : \quad V_{\phi_2}^{-1} = \Lambda_{ab} \phi_2 e^{-\varphi} \sigma_{\theta_2}^+ e^{-i(1-\theta_2)H_2} \prod_{I \neq 2}^3 \sigma_{\theta_I}^+ e^{i\theta_I H_I} \quad (2.35)$$

$$\psi_{-\frac{1}{2}+\theta_3} \mid \theta_{1,2,3} \rangle : \quad V_{\phi_3}^{-1} = \Lambda_{ab} \phi_3 e^{-\varphi} \sigma_{\theta_3}^+ e^{-i(1-\theta_3)H_3} \prod_{I \neq 3}^3 \sigma_{\theta_I}^+ e^{i\theta_I H_I}. \quad (2.36)$$

Note that in the supersymmetric case they all have  $U(1)_{WS}$  charge  $+1$  indicating that they belong to chiral super-fields together with their left-handed fermion partners.

Before turning to the second setup let us also display the vertex operators for the states  $\alpha_{\theta_1} \prod_{I=1}^3 \psi_{-\frac{1}{2}+\theta_I} \mid \theta_{1,2,3} \rangle$  and  $(\alpha_{\theta_1})^2 \prod_{I=1}^3 \psi_{-\frac{1}{2}+\theta_I} \mid \theta_{1,2,3} \rangle$

$$\begin{aligned} \alpha_{\theta_1} \prod_{I=1}^3 \psi_{-\frac{1}{2}+\theta_I} \mid \theta_{1,2,3} \rangle : \quad & V_{\Psi_{\tau_1}}^{-1} = \Lambda_{ba} \Psi_{\tau_1} e^{-\varphi} \tau_{\theta_1}^- e^{i(1-\theta_1)H_1} \prod_{I=2}^3 \sigma_{\theta_I}^- e^{i(1-\theta_I)H_I} e^{ikX} \\ (\alpha_{\theta_1})^2 \prod_{I=1}^3 \psi_{-\frac{1}{2}+\theta_I} \mid \theta_{1,2,3} \rangle : \quad & V_{\Psi_{\omega_1}}^{-1} = \Lambda_{ba} \Psi_{\omega_1} e^{-\varphi} \omega_{\theta_1}^- e^{i(1-\theta_1)H_1} \prod_{I=2}^3 \sigma_{\theta_I}^- e^{i(1-\theta_I)H_I} e^{ikX} \end{aligned}$$

Note again that the  $U(1)_{WS}$  charge dictates that these are lowest component of chiral super-fields going from brane  $b$  to brane  $a$ , as indicated by the Chan-Paton matrix.

The mass of the states is given by

$$\alpha_{\theta_1} \prod_{I=1}^3 \psi_{-\frac{1}{2}+\theta_I} | \theta_{1,2,3} \rangle \quad M^2 = \theta_1 M_s^2 \quad (2.37)$$

$$(\alpha_{\theta_1})^2 \prod_{I=1}^3 \psi_{-\frac{1}{2}+\theta_I} | \theta_{1,2,3} \rangle \quad M^2 = 2\theta_1 M_s^2 \quad (2.38)$$

which can be significantly smaller than the string scale  $M_s = 1/\sqrt{\alpha'}$ , in case the intersection angle  $\theta_1$  is very small. In section 3 we investigate whether and how in such a scenario those light states can be produced.

### Two positive angles one negative one

For the sake of concreteness we choose the third angle  $\theta_3$  to be negative. The supersymmetry condition is given by

$$\theta_1 + \theta_2 + \theta_3 = 0 . \quad (2.39)$$

The lightest states are (keep in mind that  $\theta_3$  is negative)

$$\psi_{-\frac{1}{2}+\theta_1} | \theta_{1,2,3} \rangle \quad M^2 = \frac{1}{2} (-\theta_1 + \theta_2 - \theta_3) M_s^2 \quad (2.40)$$

$$\psi_{-\frac{1}{2}+\theta_2} | \theta_{1,2,3} \rangle \quad M^2 = \frac{1}{2} (\theta_1 - \theta_2 - \theta_3) M_s^2 \quad (2.41)$$

$$\psi_{-\frac{1}{2}-\theta_3} | \theta_{1,2,3} \rangle \quad M^2 = \frac{1}{2} (\theta_1 + \theta_2 + \theta_3) M_s^2 \quad (2.42)$$

$$\psi_{-\frac{1}{2}+\theta_1} \psi_{-\frac{1}{2}+\theta_2} \psi_{-\frac{1}{2}-\theta_3} | \theta_{1,2,3} \rangle \quad M^2 = \left( 1 - \frac{1}{2} (\theta_1 + \theta_2 - \theta_3) \right) M_s^2 . \quad (2.43)$$

Again we see that for the supersymmetric condition one scalar becomes massless. The corresponding vertex operator is given by

$$\psi_{-\frac{1}{2}-\theta_3} | \theta_{1,2,3} \rangle : \quad V_{\phi_3}^{(-1)} = \Lambda_{ab} \phi_3 e^{-\varphi} \prod_{I=1}^2 \sigma_{\theta_I}^+ e^{i\theta_I H_i} \sigma_{-\theta_3}^- e^{i(1+\theta_3)H_3} e^{ikX} \quad (2.44)$$

This indeed describes the lowest component of a chiral superfield since the  $U(1)_{WS}$  charge is +1. The vertex operator of the corresponding anti-particle is given by

$$V_{\phi_3}^{(-1)} = \Lambda_{ba} \phi_3^* e^{-\varphi} \prod_{I=1}^2 \sigma_{\theta_I}^- e^{-i\theta_I H_i} \sigma_{-\theta_3}^+ e^{-i(1+\theta_3)H_3} e^{ikX} \quad (2.45)$$

and as can be easily shown has  $U(1)_{WS}$  charge -1.

With this we finish this section and turn to the R-sector.

## 2.2 R-sector

First notice that the bosonic sector is exactly the same for the R-sector as for the NS-sector. Therefore we will only focus on the fermionic one. The mode expansion of  $\Psi^I$  and  $\bar{\Psi}^I$  are similar to the expansions in the NS sector however the sum is over integers and not half-integers (see eq. (2.6))

$$\Psi^I(z) = \sum_{n \in \mathbb{Z}} \psi_{n-\theta_I}^I z^{-n-\frac{1}{2}+\theta_I} \quad \bar{\Psi}^I(z) = \sum_{n \in \mathbb{Z}} \psi_{n+\theta_I}^I \bar{z}^{-n-\frac{1}{2}-\theta_I} . \quad (2.46)$$

Again the actions of  $\Psi$  and  $\bar{\Psi}$  on the ground-state depend on whether the intersection angle is positive or negative; again we start with the scenario of a positive intersection angle.

### Positive intersection angle

In that case the ground-state is defined as

$$\psi_{n-\theta_I}^I |\theta_I\rangle = 0 \quad n \geq 1 \quad \psi_{n+\theta_I}^I |\theta_I\rangle = 0 \quad n \geq 0 . \quad (2.47)$$

Again we can compute the energy of the ground-state and with this ground-state definition it turns out to be

$$\epsilon_0 = 0 . \quad (2.48)$$

As we see momentarily it is due to this fact that at the intersection of two D-branes there is always a massless fermion independently of the intersection angles. Since we will be later only interested in the massless fermions it is sufficient to discuss only the action of  $\Psi^I$  and  $\bar{\Psi}^I$  on the ground-state  $|\theta_I\rangle$  which correspond to the spin field  $S_{\theta_I}^+(0)|0\rangle_u$ .

$$\begin{aligned} \Psi^I(z) |\theta_I\rangle &= \sum_n \psi_{n-\theta_I}^I z^{-n-\frac{1}{2}+\theta_I} |\theta_I\rangle \longrightarrow z^{-\frac{1}{2}+\theta_I} \psi_{-\theta_I}^I |\theta_I\rangle = z^{-\frac{1}{2}+\theta_I} T_{\theta}^+(0) |0\rangle_u \\ \bar{\Psi}^I(z) |\theta_I\rangle &= \sum_n \psi_{n+\theta_I}^I z^{-n-\frac{1}{2}-\theta_I} |\theta_I\rangle \longrightarrow z^{\frac{1}{2}-\theta_I} \psi_{-1+\theta_I}^I |\theta_I\rangle = z^{\frac{1}{2}-\theta_I} \tilde{T}_{\theta}^+(0) |0\rangle_u \end{aligned}$$

Again we can bosonize these fermionic twist fields which then take the form

$$\begin{aligned} S_{\alpha}^+ &= e^{i(\alpha-\frac{1}{2})H_I} & h_{S_{\alpha}^+} &= \frac{1}{2} \left( \alpha - \frac{1}{2} \right)^2 \\ T_{\alpha}^+ &= e^{i(\alpha+\frac{1}{2})H_I} & h_{T_{\alpha}^+} &= \frac{1}{2} \left( \alpha + \frac{1}{2} \right)^2 \\ \tilde{T}_{\alpha}^+ &= e^{i(\alpha-\frac{3}{2})H_I} & h_{\tilde{T}_{\alpha}^+} &= \frac{1}{2} \left( \alpha - \frac{3}{2} \right)^2 , \end{aligned} \quad (2.49)$$

where we give on the right-hand side also the conformal dimension of the respective field.

## Negative intersection angle

In such a setup the ground-state is defined as

$$\psi_{n-\theta_I}^I |\theta_I\rangle = 0 \quad n \geq 0 \quad \psi_{n+\theta_I}^I |\theta_I\rangle = 0 \quad n \geq 1 . \quad (2.50)$$

Again the zero point energy computes to zero,  $\epsilon_0 = 0$ .

The action on the ground-state  $|\theta_I\rangle$ , which corresponds to  $S_{-\theta_I}^-(0)|0\rangle$ , read

$$\begin{aligned} \Psi^I(z) |\theta_I\rangle &= \sum_n \psi_{n-\theta_I}^I z^{-n-\frac{1}{2}+\theta_I} |\theta_I\rangle \longrightarrow z^{\frac{1}{2}+\theta_I} \psi_{-1-\theta_I}^I |\theta_I\rangle = z^{\frac{1}{2}+\theta_I} \tilde{T}_{-\theta}^-(0) |0\rangle_u \\ \bar{\Psi}^I(z) |\theta_I\rangle &= \sum_n \psi_{n+\theta_I}^I z^{-n-\frac{1}{2}-\theta_I} |\theta_I\rangle \longrightarrow z^{-\frac{1}{2}-\theta_I} \psi_{\theta_I}^I |\theta_I\rangle = z^{-\frac{1}{2}-\theta_I} T_{-\theta_I}^-(0) |0\rangle_u \end{aligned}$$

Again we give below the bosonized form of the conformal fields as well as their conformal dimensions

$$\begin{aligned} S_{\alpha}^{-} &= e^{i(\alpha+\frac{1}{2})H_I} & h_{S_{\alpha}^{-}} &= \frac{1}{2} \left( \alpha - \frac{1}{2} \right)^2 \\ \tilde{T}_{\alpha}^{-} &= e^{i(\alpha+\frac{3}{2})H_I} & h_{\tilde{T}_{\alpha}^{-}} &= \frac{1}{2} \left( \alpha + \frac{3}{2} \right)^2 \\ T_{\alpha}^{-} &= e^{i(\alpha-\frac{1}{2})H_I} & h_{T_{\alpha}^{-}} &= \frac{1}{2} \left( \alpha - \frac{1}{2} \right)^2 . \end{aligned} \quad (2.51)$$

### 2.2.1 Vertex operators in the R-sector

Again we distinguish between the two different scenarios

- $\theta_1, \theta_2, \theta_3 \geq 0$  with  $\sum_I \theta_I = 2$
- $\theta_1, \theta_2 \geq 0$  and  $\theta_3 \leq 0$  with  $\sum_I \theta_I = 0$ ,

starting with the setup with all intersection angles being positive.

#### All angles positive

Since the zero point energy is zero,  $\epsilon_0 = 0$ , the ground-state  $|\theta_{1,2,3}\rangle$  describes a massless fermion.

$$|\theta_{1,2,3}\rangle : \quad V_{\bar{\psi}}^{-\frac{1}{2}} = \Lambda_{ab} \bar{\psi}_{\dot{\alpha}} e^{-\varphi/2} S^{\dot{\alpha}} \prod_{I=1}^3 \sigma_{\theta_I}^+ e^{i(\theta_I-\frac{1}{2})H_I} e^{ikX} \quad (2.52)$$

The appearance of the anti-chiral spin field  $S^{\dot{\alpha}}$  is dictated by the GSO-projection. Note that the  $U(1)_{WS}$  charge

$$\sum_{I=1}^3 \left( \theta_I - \frac{1}{2} \right) = \frac{1}{2} \quad (2.53)$$

suggests that this field is identified with a right-handed fermion belonging to an anti-chiral multiplet.

The conjugate left-handed fermion is identified with the string going from D6-brane  $b$  to D6-brane  $a$  and its vertex operator takes the form

$$|\theta_{1,2,3}\rangle : \quad V_\psi^{-\frac{1}{2}} = \Lambda_{ba} \psi_\alpha e^{-\varphi/2} S^\alpha \prod_{I=1}^3 \sigma_{-\theta_I}^- e^{i(-\theta_I + \frac{1}{2})H_I} e^{ikX} \quad (2.54)$$

Note that the  $U(1)_{WS}$  charge for this vertex operator is  $-\frac{1}{2}$  indicating that it belongs to a chiral multiplet. This vertex operator is indeed the supersymmetric partner of (2.33). Recall that the supercharge is given by

$$Q^\alpha = e^{-\varphi/2} S^\alpha \prod_{I=1}^3 e^{-\frac{i}{2}H_I} . \quad (2.55)$$

### Two positive angles one negative

Let us turn to the second setup where we choose again for the sake of concreteness  $\theta_1, \theta_2 \geq 0$  and  $\theta_3 \leq 0$ . In that case the vertex operator corresponding to the ground-state  $|\theta_{1,2,3}\rangle$  is given by

$$|\theta_{1,2,3}\rangle : \quad V_\psi^{(-1/2)} = \Lambda_{ab} \bar{\psi}_\alpha e^{-\varphi/2} S^\alpha \prod_{I=1}^2 \sigma_{\theta_I}^+ e^{i(\theta_I - \frac{1}{2})H_I} \sigma_{-\theta_3}^- e^{i(\theta_3 + \frac{1}{2})H_3} e^{ikX} . \quad (2.56)$$

It is easy to see that the  $U(1)_{WS}$  charge is indeed  $-\frac{1}{2}$  as expected for a left-handed fermion in a chiral multiplet. Note that this is also the vertex operator one obtains after applying the supercharge (2.55) to the bosonic vertex operator (2.44). The corresponding anti-particle is a string stretched from brane  $b$  to  $a$  and its vertex operator takes the form

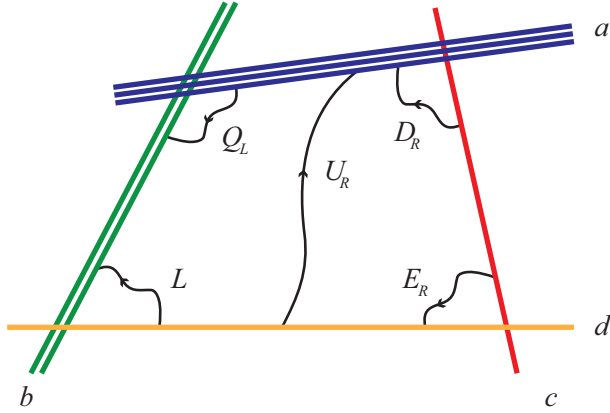
$$V_\psi^{(-1/2)} = \Lambda_{ba} \bar{\psi}_{\dot{\alpha}} e^{-\varphi/2} S^{\dot{\alpha}} \prod_{I=1}^2 \sigma_{\theta_I}^- e^{-i(\theta_I - \frac{1}{2})H_I} \sigma_{-\theta_3}^+ e^{-i(\theta_3 + \frac{1}{2})H_3} e^{ikX} . \quad (2.57)$$

## 3. Amplitudes, their factorization and all that

In the previous section we analyzed the configuration of two D6-branes intersecting at non-trivial angles. We gave a recipe for finding to any physical state the corresponding vertex operator. Moreover, we saw that there exists a tower of physical states whose mass is proportional to  $M^2 \sim \theta M_s^2$ , where  $\theta$  is the intersection angle in one of the complex dimensions and  $M_s$  is the string scale. If this product is small such states can be light and in this section we address the question whether these states can be seen and what their potential signals are.

Before we turn to that issue let us briefly recall the main features of intersecting brane worlds [1–4]. In these string compactifications, the gauge groups arise from





**Figure 1:** The Madrid quiver

stacks of D6-branes that fill out four-dimensional spacetime and wrap three-cycles in the internal Calabi-Yau threefold. Chiral matter appears at the intersection in the internal space of different cycles wrapped by the D6-brane stacks. The multiplicity of chiral matter between two stacks of D6-branes is given by the topological intersection number of the respective three-cycles.

Since many features of a D-brane compactifications, such as chiral matter, gauge symmetry or Yukawa couplings do not crucially depend on the details of the compactification, but rather on the local structure of the D-brane configurations it is often times convenient to investigate a local D-brane setup, called quiver, and to postpone the embedding into global setting. This approach is called bottom-up approach and has been initiated in [36, 37]<sup>4</sup>.

Let us present the Madrid-quiver as a concrete example of a promising local D-brane configuration that mimics the MSSM [42–44]. It consists of four D-brane stacks giving rise to the gauge symmetry  $U(3)_a \times U(2)_b \times U(1)_c \times U(1)_d$ . Generically the abelian symmetries are anomalous and get promoted to global symmetries which have to be preserved by perturbative quantities. In the Madrid quiver the linear combination

$$U(1)_Y = \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c + \frac{1}{2}U(1)_d \quad (3.1)$$

remains massless and is identified with the hypercharge. In figure 1 we display the Madrid quiver with its matter content.

In the following analysis we have in mind such a local D-brane configuration. However, instead of looking at the whole local configuration we further zoom in and just focus on a subset of the D-brane stacks and investigate the various states localized at the intersection of two stacks. Let us further specify the setup. We

<sup>4</sup>For a systemic search of realistic MSSM D-brane quivers, see [38, 39]. For an exhaustive search of global embeddings of such quivers, see [40, 41].

have three stacks of D6-branes wrapping three-cycles on the factorizable six-torus  $T^6 = T^2 \times T^2 \times T^2$ . They intersect each other non-trivially and give rise to the following intersection angles<sup>5</sup>

$$\begin{array}{lll} \theta_{ab}^1 > 0 & \theta_{ab}^2 > 0 & \theta_{ab}^3 < 0 \\ \theta_{bc}^1 > 0 & \theta_{bc}^2 > 0 & \theta_{bc}^3 < 0 \\ \theta_{ca}^1 < 0 & \theta_{ca}^2 < 0 & \theta_{ca}^3 < 0 . \end{array} \quad (3.2)$$

At each intersection massless chiral fermions appear and, in case of a preserved supersymmetry,

$$\theta_{ab}^1 + \theta_{ab}^2 + \theta_{ab}^3 = 0 \quad (3.3)$$

$$\theta_{bc}^1 + \theta_{bc}^2 + \theta_{bc}^3 = 0 \quad (3.4)$$

$$\theta_{ca}^1 + \theta_{ca}^2 + \theta_{ca}^3 = -2 \quad (3.5)$$

even massless scalars. However we do not always have to enforce them, since the analysis applies independently of whether supersymmetry is preserved or not. Moreover, in the previous section we saw that apart from the massless matter at each intersection there are also massive states whose mass scales with the intersection angle. In scenarios of a lower string mass and small intersection angles such states can be fairly light and potentially observed at LHC or future experiments.

Here we compute the scattering amplitude of four chiral fermions

$$\langle \bar{\psi} \psi \chi \bar{\chi} \rangle , \quad (3.6)$$

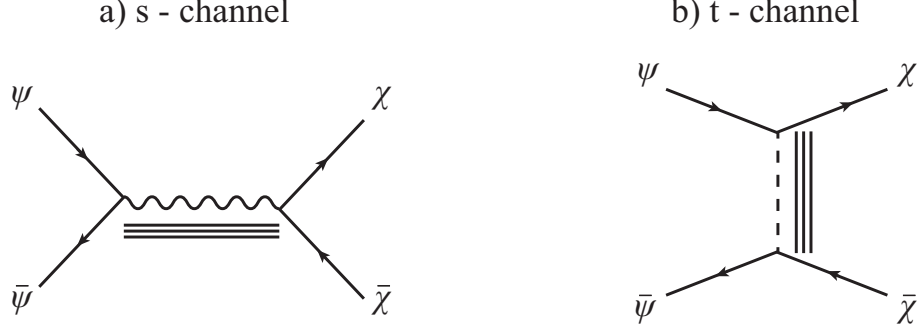
where  $\psi$  and  $\chi$  are the chiral massless fermions localized at the intersection  $ab$ , and  $bc$ , respectively. The fields  $\bar{\psi}$  and  $\bar{\chi}$  are their corresponding anti-particle. Let us discuss briefly the naive expectations concerning various limits of this amplitude.

In the  $s$ -channel, displayed in figure 2a, one expects an exchange of a gauge boson living the D-brane stack  $b$ . Indeed the dominant pole indicates a gauge boson exchange that allows one to normalize the four-point amplitude. Higher poles correspond to exchanges of stringy excitations whose mass scales with  $M_s$ . Such states can already be observed in the scattering amplitude of four gauge bosons and also in scattering of two fermions onto two gauge bosons. For a sufficient small string scale one may observe signals of these states at LHC [25, 27].

On the other hand in the  $t$ -channel, displayed in figure 2b, the dominant pole indicates the exchange of a scalar which is massless if supersymmetry is preserved. The latter is a string stretched from D6-brane  $a$  to D6-brane  $c$ . Furthermore one expects additional poles corresponding to exchanges of massive stringy states. In

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<sup>5</sup>Any other consistent choice of intersection angles is equally good, but since the CFT computation depends on the concrete form of the vertex operators, we have to make a definite choice of angles.



**Figure 2:** The s-channel: the curly line denotes the gauge boson. The t-channel: the dashed line denotes the massless scalar. The solid lines denote massive stringy states.

contrast to the  $s$ -channel exchange particles the masses of those states do not only scale with  $M_s$  but also with the intersection angle  $\theta_{ac}$ . Thus they could be significantly lighter for small intersection angle  $\theta_{ac}$  and signals of such states are expected to be observed even before observations of the massive untwisted stringy states.

### 3.1 Vertex operators

For calculating the amplitude (3.6) we need the exact form of the vertex operator. Applying the procedure laid out in section 2 to the choice of intersection angles (3.2) one obtains

$$ab : \quad V_{\psi}^{-\frac{1}{2}} = \Lambda_{ab} \psi^{\alpha} e^{-\varphi/2} S_{\alpha} \prod_{I=1}^2 \sigma_{\theta_{ab}^I}^{+} e^{i(\theta_{ab}^I - \frac{1}{2})H_I} \sigma_{-\theta_{ab}^3}^{-} e^{i(\theta_{ab}^3 + \frac{1}{2})H_3} e^{ikX} . \quad (3.7)$$

Its right-handed counterpart is given by

$$ba : \quad V_{\bar{\psi}}^{-\frac{1}{2}} = \Lambda_{ba} \bar{\psi}_{\dot{\alpha}} e^{-\varphi/2} S^{\dot{\alpha}} \prod_{I=1}^2 \sigma_{\theta_{ab}^I}^{-} e^{i(-\theta_{ab}^I + \frac{1}{2})H_I} \sigma_{-\theta_{ab}^3}^{+} e^{i(-\theta_{ab}^3 - \frac{1}{2})H_3} e^{ikX} . \quad (3.8)$$

Similarly we get for the  $bc$  sector

$$bc : \quad V_{\chi}^{-\frac{1}{2}} = \Lambda_{bc} \chi^{\alpha} e^{-\varphi/2} S_{\alpha} \prod_{I=1}^2 \sigma_{\theta_{bc}^I}^{+} e^{i(\theta_{bc}^I - \frac{1}{2})H_I} \sigma_{-\theta_{bc}^3}^{-} e^{i(\theta_{bc}^3 + \frac{1}{2})H_3} e^{ikX} . \quad (3.9)$$

Its right-handed counterpart is given by

$$cb : \quad V_{\bar{\chi}}^{-\frac{1}{2}} = \Lambda_{cb} \bar{\chi}_{\dot{\alpha}} e^{-\varphi/2} S^{\dot{\alpha}} \prod_{I=1}^2 \sigma_{\theta_{bc}^I}^{-} e^{i(-\theta_{bc}^I + \frac{1}{2})H_I} \sigma_{-\theta_{bc}^3}^{+} e^{i(-\theta_{bc}^3 - \frac{1}{2})H_3} e^{ikX} . \quad (3.10)$$

These vertex operators are sufficient for the amplitude computation (3.6) , but before turning to the computation of this amplitude let us also display the vertex operators

for the massless scalar<sup>6</sup> as well as for some light massive excitations localized at the intersection of D-branes  $a$  and  $c$ . These will be the anticipated exchange particles which are related to the dominant and sub-dominant poles in the  $t$ -channel we observe later. Here we assume that the angle  $\theta_{ca}^1$  is small, thus the lightest stringy states are generated by exciting with the bosonic operator  $\alpha_{-\theta_{ca}^1}$ .

The vertex operator for the massless scalar  $\prod_{I=1}^3 \psi_{-\frac{1}{2}-\theta_{ca}^I} |\theta_{1,2,3}\rangle$  is given by

$$V_{\phi}^{-1} = \Lambda_{ca} \phi e^{-\varphi} \prod_{I=1}^3 \sigma_{-\theta_{ca}^I}^- e^{i(1+\theta_{ca}^I)H_I} e^{ikX} \quad (3.11)$$

while the one for the first bosonic excitations takes the form

$$V_{\tilde{\phi}}^{-1} = \Lambda_{ca} \tilde{\phi} e^{-\varphi} \tau_{-\theta_{ca}^1}^- e^{i(1+\theta_{ca}^1)H_1} \prod_{I=2}^3 \sigma_{-\theta_{ca}^I}^- e^{i(1+\theta_{ca}^I)H_I} e^{ikX} \quad (3.12)$$

which corresponds to the massive state  $\alpha_{-\theta_{ca}^1} \prod_{I=1}^3 \psi_{-\frac{1}{2}-\theta_{ca}^I} |\theta_{1,2,3}\rangle$  and has mass  $M^2 = -\theta_{ca}^1 M_s^2$ . The second state we consider is  $(\alpha_{-\theta_{ca}^1})^2 \prod_{I=1}^3 \psi_{-\frac{1}{2}-\theta_{ca}^I} |\theta_{1,2,3}\rangle$ , that has mass  $M^2 = -2\theta_{ca}^1 M_s^2$  and whose vertex operator is given by

$$V_{\tilde{\tilde{\phi}}}^{-1} = \Lambda_{ca} \tilde{\tilde{\phi}} e^{-\varphi} \omega_{-\theta_{ca}^1}^- e^{i(1+\theta_{ca}^1)H_1} \prod_{I=2}^3 \sigma_{-\theta_{ca}^I}^- e^{i(1+\theta_{ca}^I)H_I} e^{ikX} . \quad (3.13)$$

It is easy to check that the conformal dimensions of these vertex operators indeed account for states with mass  $M^2 = -\theta_{ca}^1 M_s^2$  and  $M^2 = -2\theta_{ca}^1 M_s^2$ , respectively.

### 3.2 The amplitude

Given these vertex operators we are now able to compute the amplitude

$$\langle \bar{\psi}(0) \psi(x) \chi(1) \bar{\chi}(\infty) \rangle \quad (3.14)$$

that allows us to extract the Yukawa coupling between the fields  $\psi$ ,  $\chi$  and  $\phi$  (as well as  $\tilde{\phi}$  and  $\tilde{\tilde{\phi}}$ ). Plugging in the vertex operators one obtains

$$\begin{aligned} \mathcal{A} = & \text{Tr}(\Lambda_{ba} \Lambda_{ab} \Lambda_{bc} \Lambda_{cb}) \bar{\psi}_{\dot{\alpha}} \psi^{\alpha} \chi^{\beta} \bar{\chi}_{\dot{\beta}} \int_0^1 dx \left\langle e^{-\varphi/2(0)} e^{-\varphi/2(x)} e^{-\varphi/2(1)} e^{-\varphi/2(\infty)} \right\rangle \\ & \times \left\langle S^{\dot{\alpha}}(0) S_{\alpha}(x) S_{\beta}(1) S^{\dot{\beta}}(\infty) \right\rangle \left\langle e^{ik_1 X(0)} e^{ik_2 X(x)} e^{ik_3 X(1)} e^{ik_4 X(\infty)} \right\rangle \\ & \times \left\langle \sigma_{-\theta_{ab}^3}^+(0) \sigma_{-\theta_{ab}^3}^-(x) \sigma_{-\theta_{bc}^3}^-(1) \sigma_{-\theta_{bc}^3}^+(\infty) \right\rangle \prod_{I=1}^2 \left\langle \sigma_{\theta_{ab}^I}^-(0) \sigma_{\theta_{ab}^I}^+(x) \sigma_{\theta_{bc}^I}^+(0) \sigma_{\theta_{bc}^I}^-(\infty) \right\rangle \\ & \times \prod_{I=1}^2 \left\langle e^{i(-\theta_{ab}^I + \frac{1}{2})H^I(0)} e^{i(\theta_{ab}^I - \frac{1}{2})H^I(x)} e^{i(\theta_{bc}^I - \frac{1}{2})H^I(1)} e^{i(-\theta_{bc}^I + \frac{1}{2})H^I(\infty)} \right\rangle \\ & \times \left\langle e^{i(-\theta_{ab}^3 - \frac{1}{2})H^3(0)} e^{i(\theta_{ab}^3 + \frac{1}{2})H^3(x)} e^{i(\theta_{bc}^3 + \frac{1}{2})H^3(1)} e^{i(-\theta_{bc}^3 - \frac{1}{2})H^3(\infty)} \right\rangle . \end{aligned} \quad (3.15)$$

---

<sup>6</sup>The scalar is massless only when supersymmetry is preserved.

The necessary correlators are given by

$$\left\langle e^{-\varphi/2(0)} e^{-\varphi/2(x)} e^{-\varphi/2(1)} e^{-\varphi/2(\infty)} \right\rangle = [x(1-x)]^{-\frac{1}{4}} x_{\infty}^{-\frac{3}{4}} \quad (3.16)$$

$$\left\langle S^{\dot{\alpha}}(0) S_{\alpha}(x) S_{\beta}(1) S^{\dot{\beta}}(\infty) \right\rangle = \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} (1-x)^{-\frac{1}{2}} x_{\infty}^{-\frac{1}{2}} \quad (3.17)$$

$$\left\langle e^{ik_1 X(0)} e^{ik_2 X(x)} e^{ik_3 X(1)} e^{ik_4 X(\infty)} \right\rangle = x^{k_1 \cdot k_2} (1-x)^{k_2 \cdot k_3} x_{\infty}^{k_4(k_1+k_2+k_3)} \quad (3.18)$$

$$\left\langle e^{i\alpha H^I(0)} e^{i\beta H^I(x)} e^{i\gamma H^I(1)} e^{i\delta H^I(\infty)} \right\rangle = x^{\alpha\beta} (1-x)^{\beta\gamma} x_{\infty}^{\delta(\alpha+\beta+\gamma)} . \quad (3.19)$$

Finally, we also need the correlator containing four bosonic twist fields. It turns out that the following is true (see appendix A and [45])

$$\sigma_{\theta}^{-}(z) = \sigma_{1-\theta}^{+}(z) \quad \sigma_{-\theta}^{-}(z) = \sigma_{1+\theta}^{+}(z) \quad (3.20)$$

This simplifies the computation since one does not have to determine the twist field correlators for different combinations of "twist" and "anti-twist" fields separately but rather can use the result computed for one combination and appropriately plug in the appropriate angles.

The bosonic twist field correlator is given by [25, 35, 45–47]

$$x_{\infty}^{\nu(1-\nu)} \langle \sigma_{1-\theta}^{+}(0) \sigma_{\theta}^{+}(x) \sigma_{1-\nu}^{+}(1) \sigma_{\nu}^{+}(\infty) \rangle = x^{-\theta(1-\theta)} (1-x)^{-\frac{1}{2}(\theta+\nu)+\theta\nu} I^{-\frac{1}{2}}(\theta, \nu, x) e^{-S_{cl}(\theta, \nu)} , \quad (3.21)$$

with  $I(x, \theta, \nu)$  given by

$$I(\theta, \nu, x) = \frac{1}{2\pi} [B_1(\theta, \nu) G_2(x) H_1(1-x) + B_2(\theta, \nu) G_1(x) H_2(1-x)] ,$$

where

$$B_1(\theta, \nu) = \frac{\Gamma(\theta) \Gamma(1-\nu)}{\Gamma(1+\theta-\nu)} \quad B_2(\theta, \nu) = \frac{\Gamma(\nu) \Gamma(1-\theta)}{\Gamma(1+\nu-\theta)}$$

$$G_1(x) = {}_2F_1[\theta, 1-\nu, 1; x] \quad G_2(x) = {}_2F_1[1-\theta, \nu, 1; x]$$

$$H_1(x) = {}_2F_1[\theta, 1-\nu, 1+\theta-\nu; x] \quad H_2(x) = {}_2F_1[1-\theta, \nu, 1-\theta+\nu; x] .$$

and the classical contribution<sup>7</sup>

$$e^{-S_{cl}(\theta, \nu)} = \sum_{\tilde{p}_i, q_i} \exp \left[ -\pi \frac{\sin(\pi\theta)}{t(\theta, \nu, x)} \frac{L_{bi}^2}{\alpha'} \tilde{p}_i^2 - \pi \frac{t(\theta, \nu, x)}{\sin(\pi\theta)} \frac{R_{xi}^2 R_{yi}^2}{\alpha' L_{bi}^2} q_i^2 \right] \quad (3.22)$$

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<sup>7</sup>For the sake of clarity here we simplify the configuration by assuming that all three D-branes are intersecting exactly once and all Wilson lines are vanishing. A generalization of the results can be easily obtained using the results of [25, 35, 44, 48]

with  $t(\theta, \nu, x)$  given by

$$t(\theta, \nu, x) = \frac{\sin(\pi\theta)}{2\pi} \left( \frac{B_1 H_1(1-x)}{G_1(x)} + \frac{B_2 H_2(1-x)}{G_2(x)} \right) . \quad (3.23)$$

Combining the various correlators and taking into account the c-ghost contribution  $\langle c(0)c(1)c(\infty) \rangle = x_\infty^{-2}$  one obtains for the amplitude

$$\begin{aligned} \mathcal{A} \sim i g_s \text{Tr} (\Lambda_{ba} \Lambda_{ab} \Lambda_{bc} \Lambda_{cb}) \bar{\psi} \cdot \bar{\chi} \psi \cdot \chi (2\pi)^4 \delta^{(4)} \left( \sum_i^4 k_i \right) \\ \times \int_0^1 dx \frac{x^{-1+k_1 \cdot k_2} (1-x)^{-\frac{3}{2}+k_2 \cdot k_3} e^{-S_{cl}(\theta_{ab}^1, 1-\theta_{bc}^1)} e^{-S_{cl}(\theta_{ab}^2, 1-\theta_{bc}^2)} e^{-S_{cl}(1+\theta_{ab}^3, -\theta_{bc}^3)}}{[I(\theta_{ab}^1, 1-\theta_{bc}^1, x) I(\theta_{ab}^2, 1-\theta_{bc}^2, x) I(1+\theta_{ab}^3, -\theta_{bc}^3, x)]^{\frac{1}{2}}} \end{aligned} \quad (3.24)$$

### 3.3 s-channel – normalization of the amplitude

Before turning to the t-channel, where we expect the exchange of light stringy states, we will investigate the s-channel which allows us to normalize the amplitude. In order to properly take the limit  $x \rightarrow 0$  we Poisson resum the classical contribution, obtaining

$$\begin{aligned} \mathcal{A} \sim i g_s \text{Tr} (\Lambda_{ba} \Lambda_{ab} \Lambda_{bc} \Lambda_{cb}) \bar{\psi} \cdot \bar{\chi} \psi \cdot \chi \frac{(2\pi)^4 \delta^{(4)} (\sum_i^4 k_i)}{L_{b^1} L_{b^2} L_{b^3}} \\ \times \int_0^1 dx \frac{x^{-1+k_1 \cdot k_2} (1-x)^{-\frac{3}{2}+k_2 \cdot k_3} e^{-\tilde{S}_{cl}(\theta_{ab}^1, 1-\theta_{bc}^1)} e^{-\tilde{S}_{cl}(\theta_{ab}^2, 1-\theta_{bc}^2)} e^{-\tilde{S}_{cl}(1+\theta_{ab}^3, -\theta_{bc}^3)}}{\sqrt{{}_2F_1[\theta_{ab}^1, \theta_{bc}^1, 1; x] {}_2F_1[\theta_{ab}^2, \theta_{bc}^2, 1; x] {}_2F_1[1+\theta_{ab}^3, 1+\theta_{bc}^3, 1; x]}}, \end{aligned} \quad (3.25)$$

where  $e^{-\tilde{S}_{cl}}$  in the Lagrangian form is given by

$$e^{-\tilde{S}_{cl}(\theta, \nu)} = \prod_{i=1}^3 \sum_{p_i, q_i} \exp \left[ -\pi \frac{t(\theta, \nu, x)}{\sin(\pi\theta)} \frac{\alpha'}{L_{b_i}^2} p_i^2 - \pi \frac{t(\theta, \nu, x)}{\sin(\pi\theta)} \frac{R_{x_i}^2 R_{y_i}^2}{\alpha' L_{b_i}^2} q_i^2 \right] . \quad (3.26)$$

In the limit  $x \rightarrow 0$  that corresponds to the s-channel one has

$$t(\theta, \nu, x) \approx \frac{\sin(\pi\theta)}{\pi} (-\ln(x) + \ln(\delta)) \quad (3.27)$$

with  $\ln(\delta)$  given by

$$\ln(\delta) = 2\psi(1) - \frac{1}{2} (\psi(\theta) + \psi(1-\theta) + \psi(\nu) + \psi(1-\nu)) . \quad (3.28)$$

Thus the dominant pole in the s-channel is

$$\begin{aligned} \mathcal{A} = i g_s \mathcal{C} \text{Tr} (\Lambda_{ba} \Lambda_{ab} \Lambda_{bc} \Lambda_{cb}) (2\pi)^4 \delta^{(4)} \left( \sum_i^4 k_i \right) \bar{\psi} \cdot \bar{\chi} \psi \cdot \chi \\ \times \frac{\alpha'^{\frac{3}{2}}}{L_{b^1} L_{b^2} L_{b^3}} \int_0^{0+\epsilon} dx x^{-1+s} \prod_{i=1}^3 \sum_{p_i, q_i} \left( \frac{x}{\delta} \right)^{\frac{\alpha'}{L_{b_i}^2} p_i^2 + \frac{R_{x_i}^2 R_{y_i}^2}{\alpha' L_{b_i}^2} q_i^2} . \end{aligned} \quad (3.29)$$

For  $p_i = q_i = 0$  the amplitude factorizes on the exchange of gauge bosons

$$A_4(k_1, k_2, k_3, k_4) = i \int \frac{d^4 k d^4 k'}{(2\pi)^4} \frac{\sum_g A_\mu^g(k_1, k_2, k) A^{g,\mu}(k_3, k_4, k') \delta^{(4)}(k - k')}{k^2 - i\epsilon} . \quad (3.30)$$

Knowing the form of the three point amplitude allows us to normalize the amplitude. In eq (3.30) we sum over all polarizations (vector index  $\mu$ ) and all colors (adjoint index  $g$ ) that can be exchanged. The three-point amplitude describing the coupling of the two fermions to a gauge boson is given by [30]

$$A_\mu^g(k_1, k_2, k_3) = i g_{D6_b} (2\pi)^4 \delta^{(4)} \left( \sum_{i=1}^3 k_i \right) \bar{\psi} \sigma^\mu \psi \text{Tr}(\Lambda_{ba} \Lambda_{ab} \Lambda_{bb}) . \quad (3.31)$$

Here  $\Lambda_{bb}$  denotes the Chan-Paton matrix of the exchanged gauge boson and the gauge coupling [49]  $g_{D6_b}^2 = (2\pi)^4 \alpha'^{3/2} g_s / \prod_{i=1}^3 2\pi L_{b_i}$ . Performing the integral (3.30) and comparing with (3.29) gives for the normalization

$$\mathcal{C} = 2\pi . \quad (3.32)$$

Here we used the usual normalization  $\text{Tr}(\lambda_a \lambda_b) = \frac{1}{2} \delta_{ab}$ .

Non-vanishing  $p_i$  and  $q_i$  in (3.29) indicate exchanges of KK and winding states, respectively. The exchanges of these states probe the geometry of the D-brane configuration and thus are very model-dependent. On the other hand there are higher order poles not originating from the world-sheet contributions that are related to stringy excitations. Including sub-dominant terms of the hypergeometric functions in the limit  $x \rightarrow 0$  gives

$$\begin{aligned} \mathcal{A} = & 2i\pi g_s \text{Tr}(\Lambda_{ba} \Lambda_{ab} \Lambda_{bc} \Lambda_{cb}) (2\pi)^4 \delta^{(4)} \left( \sum_i^4 k_i \right) \bar{\psi} \cdot \bar{\chi} \psi \cdot \chi \\ & \times \frac{\alpha'^{\frac{3}{2}}}{L_{b1} L_{b2} L_{b3}} \int_0^{0+\epsilon} dx x^{-1+s} (1 + c_1 x + c_2 x^2 + \dots) \prod_{i=1}^3 \sum_{p_i, q_i} \left( \frac{x}{\delta} \right)^{\frac{\alpha'}{L_{b_i}^2} p_i^2 + \frac{R_{x_i}^2 R_{y_i}^2}{\alpha' L_{b_i}^2} q_i^2} . \end{aligned} \quad (3.33)$$

where  $c_i$  are angle dependent coefficients. Note that the sub-dominant poles are integer modded indicating that the mass of the exchanged particles is of order  $M_s$ , and can be potentially observed at LHC if the string scale is in the  $TeV$  range [8, 10]. However the signals are very similar to the ones observed in the scattering of multiple gauge bosons onto at most two fermions which have been investigated in [25, 27, 28, 50].

### 3.4 t-channel– exchange of light stringy states

In this channel we expect the exchange of a massless scalar in case of preserved supersymmetry as well as additional massive states whose mass is basically given by the product of the intersection angle and the string scale  $M_s$ . If the intersection

angle is small these will be long-lived resonances which in case of a low string scale could be observed at LHC. In addition to these light-stringy excitations one can also observe exchanges of massive stringy states that even in the limit of a vanishing intersection angle remain massive. We will briefly comment on those resonances.

In order to perform this analysis we have to determine the behaviour of  $I(\theta, \nu, x)$  and  $t(\theta, \nu, x)$  in the limit  $x \rightarrow 1$ . Using the properties of the hypergeometric functions displayed in appendix B one obtains for  $I(\theta, \nu, x)$

$$\lim_{x \rightarrow 0} \frac{1}{2\pi} I(\theta, \nu, x) \sim \frac{\Gamma(1-\theta) \Gamma(\nu) \Gamma(1+\theta-\nu)}{\Gamma(\theta) \Gamma(1-\nu) \Gamma(\nu-\theta)} (1-x)^{\theta-\nu} + \frac{\Gamma(\theta) \Gamma(1-\nu) \Gamma(1-\theta+\nu)}{\Gamma(1-\theta) \Gamma(\nu) \Gamma(\theta-\nu)} (1-x)^{\nu-\theta} \quad (3.34)$$

and for  $t(\theta, \nu, x)$  we distinguish among two different scenarios, depending on which angle is larger

$$\lim_{x \rightarrow 1} t(\theta, \nu, x) = \frac{\sin(\pi(\theta-\nu))}{2 \sin(\pi\nu)} \quad \text{for } \theta > \nu \quad (3.35)$$

$$\lim_{x \rightarrow 1} t(\theta, \nu, x) = \frac{\sin(\pi(\nu-\theta))}{2 \sin(\pi\nu)} \quad \text{for } \theta < \nu. \quad (3.36)$$

With this the amplitude behaves

$$\begin{aligned} \mathcal{A} = & 2i\pi g_s \text{Tr} (\Lambda_{ba} \Lambda_{ab} \Lambda_{bc} \Lambda_{cb}) \bar{\psi} \cdot \bar{\chi} \psi \cdot \chi (2\pi)^4 \delta^{(4)} \left( \sum_i^4 k_i \right) \int_{1-\epsilon}^1 dx (1-x)^{-\frac{3}{2}+k_2 \cdot k_3} \\ & \times \left[ \left( \Gamma_{1-\theta_{ab}^1, 1-\theta_{bc}^1, \theta_{ab}^1+\theta_{bc}^1} (1-x)^{\theta_{ab}^1+\theta_{bc}^1-1} + \Gamma_{\theta_{ab}^1, \theta_{bc}^1, 2-\theta_{ab}^1-\theta_{bc}^1} (1-x)^{1-\theta_{ab}^1-\theta_{bc}^1} \right) \right]^{-\frac{1}{2}} \\ & \times \left[ \left( \Gamma_{1-\theta_{ab}^2, 1-\theta_{bc}^2, \theta_{ab}^2+\theta_{bc}^2} (1-x)^{\theta_{ab}^2+\theta_{bc}^2-1} + \Gamma_{\theta_{ab}^2, \theta_{bc}^2, 2-\theta_{ab}^2-\theta_{bc}^2} (1-x)^{1-\theta_{ab}^2-\theta_{bc}^2} \right) \right]^{-\frac{1}{2}} \\ & \times \left[ \left( \Gamma_{-\theta_{ab}^3, -\theta_{bc}^3, 2+\theta_{ab}^3+\theta_{bc}^3} (1-x)^{1+\theta_{ab}^3+\theta_{bc}^3} + \Gamma_{1+\theta_{ab}^3, 1+\theta_{bc}^3, -\theta_{ab}^3-\theta_{bc}^3} (1-x)^{-\theta_{ab}^3-\theta_{bc}^3-1} \right) \right]^{-\frac{1}{2}} \\ & \times \prod_{p_i, q_i}^2 e^{-S_{cl}^3(\theta_{ab}^i, 1-\theta_{bc}^i, p_i)} e^{-S_{cl}^3(\theta_{ab}^i, 1-\theta_{bc}^i, q_i)} e^{-S_{cl}^3(1+\theta_{ab}^3, -\theta_{bc}^3, p_3)} e^{-S_{cl}^3(1+\theta_{ab}^3, -\theta_{bc}^3, q_3)}, \end{aligned} \quad (3.37)$$

where  $\Gamma_{\alpha, \beta, \gamma}$  is given by

$$\Gamma_{\alpha, \beta, \gamma} = \frac{\Gamma(\alpha) \Gamma(\beta) \Gamma(\gamma)}{\Gamma(1-\alpha) \Gamma(1-\beta) \Gamma(1-\gamma)} \quad (3.38)$$

and  $e^{-S_{cl}^3(\theta, \nu, p)}$  takes the form [45]<sup>8</sup>

$$e^{-S_{cl}^3(\theta, \nu, p_i)} = \exp \left[ -\frac{\pi \sin(\pi\theta) \sin(\pi\nu)}{4 |\sin(\pi(\theta-\nu))|} \frac{L_{b_i}}{\alpha'} p_i^2 \right]. \quad (3.40)$$

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<sup>8</sup>Recall that all three branes intersect exactly once and for simplicity we assume vanishing Wilson



To simplify the analysis further let us assume that we are in the large volume limit, thus  $R_{x_i}, R_{y_i}$  are large. Thus all world-sheet instanton contributions from  $p_i, q_i \neq 0$  are negligible. Additionally for the sake of concreteness the intersection angles satisfy

$$\theta_{ab}^1 + \theta_{bc}^1 < 1 \quad \theta_{ab}^2 + \theta_{bc}^2 < 1 \quad |\theta_{ab}^3 + \theta_{bc}^3| > 1. \quad (3.41)$$

Given that we can pull out the dominant pole and get for the amplitude

$$\begin{aligned} \mathcal{A} = & 2i\pi g_s \text{Tr}(\Lambda_{ba} \Lambda_{ab} \Lambda_{bc} \Lambda_{cb}) \bar{\psi} \cdot \bar{\chi} \psi \cdot \chi (2\pi)^4 \delta^{(4)} \left( \sum_i^4 k_i \right) \\ & \times \int_{1-\epsilon}^1 dx \frac{(1-x)^{-1-\frac{1}{2}\sum_I(\theta_{ab}^I + \theta_{bc}^I) + k_2 \cdot k_3}}{\Gamma_{1-\theta_{ab}^1, 1-\theta_{bc}^1, \theta_{ab}^1 + \theta_{bc}^1}^{\frac{1}{2}} \Gamma_{1-\theta_{ab}^2, 1-\theta_{bc}^2, \theta_{ab}^2 + \theta_{bc}^2}^{\frac{1}{2}} \Gamma_{-\theta_{ab}^3, -\theta_{bc}^3, 2+\theta_{ab}^3 + \theta_{bc}^3}^{\frac{1}{2}}} \\ & \times \left[ \left(1 + c_1(1-x)^{2(1-\theta_{ab}^1 - \theta_{bc}^1)}\right) \left(1 + c_2(1-x)^{2(1-\theta_{ab}^2 - \theta_{bc}^2)}\right) \left(1 + c_3(1-x)^{2(-\theta_{ab}^3 - \theta_{bc}^3 - 1)}\right) \right]^{-\frac{1}{2}}. \end{aligned} \quad (3.42)$$

Here the  $c_i$ 's are given by

$$c_1 = \frac{\Gamma_{1-\theta_{ab}^1, 1-\theta_{bc}^1, \theta_{ab}^1 + \theta_{bc}^1}}{\Gamma_{\theta_{ab}^1, \theta_{bc}^1, 2-\theta_{ab}^1 - \theta_{bc}^1}}, \quad c_2 = \frac{\Gamma_{1-\theta_{ab}^2, 1-\theta_{bc}^2, \theta_{ab}^2 + \theta_{bc}^2}}{\Gamma_{\theta_{ab}^2, \theta_{bc}^2, 2-\theta_{ab}^2 - \theta_{bc}^2}}, \quad c_3 = \frac{\Gamma_{-\theta_{ab}^3, -\theta_{bc}^3, 2+\theta_{ab}^3 + \theta_{bc}^3}}{\Gamma_{1+\theta_{ab}^3, 1+\theta_{bc}^3, -\theta_{ab}^3 - \theta_{bc}^3}}.$$

In the supersymmetric case this amplitude further simplifies and gives

$$\begin{aligned} \mathcal{A} = & \bar{\psi} \cdot \bar{\chi} \psi \cdot \chi \int_{1-\epsilon}^1 dx (1-x)^{-1+k_2 \cdot k_3} \Gamma_{1-\theta_{ab}^1, 1-\theta_{bc}^1, \theta_{ab}^1 + \theta_{bc}^1}^{-\frac{1}{2}} \Gamma_{1-\theta_{ab}^2, 1-\theta_{bc}^2, \theta_{ab}^2 + \theta_{bc}^2}^{-\frac{1}{2}} \Gamma_{-\theta_{ab}^3, -\theta_{bc}^3, 2+\theta_{ab}^3 + \theta_{bc}^3}^{-\frac{1}{2}} \\ & \times \left[ \left(1 + c_1(1-x)^{2(1-\theta_{ab}^1 - \theta_{bc}^1)}\right) \left(1 + c_2(1-x)^{2(1-\theta_{ab}^2 - \theta_{bc}^2)}\right) \left(1 + c_3(1-x)^{2(-\theta_{ab}^3 - \theta_{bc}^3 - 1)}\right) \right]^{-\frac{1}{2}} \end{aligned}$$

The first thing to note is that one indeed observes the exchange of a massless scalar<sup>9</sup>. This particle is identified with  $\phi$  whose vertex operator is displayed in eq. (3.11).

The corresponding physical Yukawa coupling between  $\psi$ ,  $\chi$  and  $\phi$  is then

$$Y_{\psi\chi\phi} \sim \Gamma_{1-\theta_{ab}^1, 1-\theta_{bc}^1, \theta_{ab}^1 + \theta_{bc}^1}^{-\frac{1}{4}} \Gamma_{1-\theta_{ab}^2, 1-\theta_{bc}^2, \theta_{ab}^2 + \theta_{bc}^2}^{-\frac{1}{4}} \Gamma_{-\theta_{ab}^3, -\theta_{bc}^3, 2+\theta_{ab}^3 + \theta_{bc}^3}^{-\frac{1}{4}}. \quad (3.43)$$

Since the angles depend non-holomorphically on the complex structure moduli the Gamma-function expressions cannot be part of the holomorphic Yukawa couplings

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lines and a rectangular torus. With this in mind the intersection angles are given by

$$|\sin(\pi\theta_{ab}^i)| = \frac{R_1 R_2}{L_{a_i} L_{b_i}} \quad |\sin(\pi\theta_{bc}^i)| = \frac{R_1 R_2}{L_{c_i} L_{a_i}} \quad |\sin(\pi(\theta_{ab}^i - \theta_{bc}^i))| = \frac{R_1 R_2}{L_{b_i} L_{c_i}}. \quad (3.39)$$

For a generalization to setups with non-vanishing Wilson lines and multiple intersections among the three D-branes, see [35, 44, 46, 48].

<sup>9</sup>In the non-susy case the lightest exchange particle has mass  $M^2 = \frac{1}{2} \sum_{I=1}^3 (\theta_{ab}^I + \theta_{bc}^I)$ .

but should rather arise from the Kähler potential. Thus it is natural to assume that the Kähler metrics take the form

$$\mathcal{K}_\psi \sim \left( \frac{\Gamma(\theta_{ab}^1)}{\Gamma(1-\theta_{ab}^1)} \frac{\Gamma(\theta_{ab}^2)}{\Gamma(1-\theta_{ab}^2)} \frac{\Gamma(-\theta_{ab}^3)}{\Gamma(1-\theta_{ab}^3)} \right)^{\frac{1}{4}} \quad (3.44)$$

$$\mathcal{K}_\chi \sim \left( \frac{\Gamma(\theta_{bc}^1)}{\Gamma(1-\theta_{bc}^1)} \frac{\Gamma(\theta_{bc}^2)}{\Gamma(1-\theta_{bc}^2)} \frac{\Gamma(-\theta_{bc}^3)}{\Gamma(1-\theta_{bc}^3)} \right)^{\frac{1}{4}} \quad (3.45)$$

$$\mathcal{K}_\phi \sim \left( \frac{\Gamma(-\theta_{ca}^1)}{\Gamma(1+\theta_{ca}^1)} \frac{\Gamma(-\theta_{ca}^2)}{\Gamma(1+\theta_{ca}^2)} \frac{\Gamma(-\theta_{ca}^3)}{\Gamma(1+\theta_{ca}^3)} \right)^{\frac{1}{4}} \quad (3.46)$$

which is in complete agreement with previous derivations [29, 47].

Let us investigate sub-dominant poles of this amplitude. Recall that we expect massive scalar exchanges, whose mass scales as  $M^2 \sim \theta_{ca}^I M_s^2$ . The expansion  $x \rightarrow 1$ , including sub-dominant poles gives

$$\begin{aligned} & \left[ \left( 1 + c_1(1-x)^{2(1-\theta_{ab}^1-\theta_{bc}^1)} \right) \left( 1 + c_2(1-x)^{2(1-\theta_{ab}^2-\theta_{bc}^2)} \right) \left( 1 + c_3(1-x)^{2(-\theta_{ab}^3-\theta_{bc}^3-1)} \right) \right]^{-\frac{1}{2}} \\ & \simeq 1 + c_1(1-x)^{2(1-\theta_{ab}^1-\theta_{bc}^1)} + c_2(1-x)^{2(1-\theta_{ab}^2-\theta_{bc}^2)} + c_3(1-x)^{2(-\theta_{ab}^3-\theta_{bc}^3-1)} + \dots \end{aligned} \quad (3.47)$$

For concreteness we assume that  $1 - \theta_{ab}^1 - \theta_{bc}^1 = -\theta_{ca}^1$  is small.

Then the amplitude takes the following form

$$\mathcal{A} = \bar{\psi} \cdot \bar{\chi} \psi \cdot \chi \int_{1-\epsilon}^1 dx (1-x)^{-1+k_2 \cdot k_3} Y_{\psi\chi\phi}^2 \left( 1 + c_1(1-x)^{2(1-\theta_{ab}^1-\theta_{bc}^1)} + \dots \right), \quad (3.48)$$

the first sub-dominant term suggests that there is a particle with mass  $M^2 = -2\theta_{ca}^1$  exchanged.

As we have discussed in the beginning of this section, the spectrum in the  $ca$  sector indeed reveals a particle with the mass  $-2\theta_{ca}^1 M_s^2$ , namely the scalar  $\tilde{\phi}$ , whose vertex operator is given in eq. (3.13). Let us stress that there is no coupling to the lightest massive field  $\tilde{\phi}$ , which one naively may expect. This is due to the fact that the two bosonic twist fields  $\sigma$  do not couple to the excited twist field  $\tau$ , but they only couple to an even excited twist field [45]. In agreement with the latter an inspection of higher poles reveals that the next lightest state exchanged has a mass of  $-4\theta_{ca}^1 M_s^2$ . A detailed analysis of the next-lighter massive states while straight-forward is beyond the scope of the present investigation. Similarly we do not analyze massive states, whose masses do not vanish for small angles, but we expect similar results as derived in [24, 25, 27, 28, 50]. Such an analysis would require a more detailed analysis of the sub-dominant poles of the hypergeometric functions. Note that while signals induced by light stringy states at colliders could be rather difficult to recognize and discriminate from other kinds of Physics Beyond the Standard Model, still these signals are expected to be observed first. Moreover, at higher energy scales one eventually will observe higher spin state signatures, which then hint towards a stringy nature.

## 4. Summary and Conclusions

Let us conclude by summarizing our results and drawing some lines for future investigation.

We have carefully studied the spectrum of open strings localized at the intersections of D6-branes. At the cost of being pedantic and partially overlapping with previous investigations [25, 27, 30], we have identified the ground-states as well as the lowest massive states and displayed the corresponding vertex operators both in the NS- and R-sectors. We had to pay particular attention to the signs of the intersection angles [25, 35, 46, 51] since the relevant twist fields depend crucially on those. We have also checked the presence of massless scalars when the angles satisfy supersymmetry preserving conditions. We have argued that the masses of the lightest states scale as  $M_\theta^2 \approx \theta M_s^2$  and can thus be parametrically smaller than the string scale if the relevant angle is small. This in turn depends both on the wrapping numbers of the D6-branes and the shape of the tori or orbifolds. We have not address the issue of (supersymmetric) moduli stabilization, which is still open – at least from a world-sheet CFT vantage point – and seems to be in tension with chirality. Instead we have considered processes that can expose these light stringy states in their intermediate channel. Relying on previous analysis, we have computed 4-point scattering amplitudes of ‘twisted’ open strings and studied their factorization in the s- and t-channel confirming the presence of the sought for states as sub-dominant poles in the latter.

Although straightforward we have not analyzed the poles corresponding to massive, possibly higher spin, states which remain massive even when some angles are small. Their analysis is tedious and presents significant analogies with the analysis in [25, 27, 28, 50]. Notwithstanding the limitations of our analysis, we cannot help drawing some phenomenological conclusions. Assuming a scenario with large extra dimensions and a low scale string tension proves to be realized in Nature, the spectrum of string excitations may be rather ‘irregular’ or at least look very different to the regularly spaced Regge recurrences of the good old Veneziano model. Signals at colliders could be rather difficult to recognize and discriminate from other kinds of Physics Beyond the Standard Model. Yet, the possibility that the lightest massive string excitations be just behind the corner makes worth sharpening our predictions and/or generalizing it to phenomenologically more viable models, possibly including the effect of closed string fluxes and non-perturbative effects.

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## A. The twist and anti-twist fields $\sigma_\theta^+$ and $\sigma_\theta^-$

Let us take a look at the OPE of these two fields with the conformal fields  $\partial Z^I$  and  $\partial \bar{Z}^I$ . As shown in section 2.1 we have the following OPE's

$$\partial Z^I(z) \sigma_\theta^+(w) \sim (z-w)^{\theta-1} \tau_\theta^+(w) \quad \partial \bar{Z}^I(z) \sigma_\theta^+(w) \sim (z-w)^{-\theta} \tilde{\tau}_\theta^+(w) \quad (\text{A.1})$$

$$\partial Z^I(z) \sigma_\theta^-(w) \sim (z-w)^\theta \tilde{\tau}_\theta^-(w) \quad \partial \bar{Z}^I(z) \sigma_\theta^-(w) \sim (z-w)^{\theta-1} \tau_\theta^-(w) \quad (\text{A.2})$$

Their conformal dimensions are given by

$$h_{\sigma_\theta^+} = \frac{1}{2}\theta(1-\theta) \quad h_{\sigma_\theta^-} = -\frac{1}{2}\theta(1+\theta) \quad (\text{A.3})$$

which suggests the following identification

$$\sigma_\theta^- = \sigma_{1+\theta}^+ . \quad (\text{A.4})$$

Analogously one can also show that

$$\sigma_{-\theta}^- = \sigma_{1-\theta}^+ . \quad (\text{A.5})$$

For more details on these identifications, specifically in the context of excited twist fields, see [45].

## B. Properties of hypergeometric functions

In this appendix we display various properties of hypergeometric functions that we will use throughout the paper.

The hypergeometric function is given by

$${}_2F_1[\theta, 1-\nu, 1, z] = \frac{1}{\Gamma(\theta)\Gamma(1-\nu)} \sum_{n=0}^{\infty} \frac{\Gamma(\theta+n)\Gamma(1-\nu+n)}{\Gamma(n)} \frac{z^n}{n!} . \quad (\text{B.1})$$

where the series is only convergent for  $|z| \leq 1$ . Below we display some relations of the hypergeometric functions, starting with

$${}_2F_1[a, b, c, z] = (1-z)^{c-a-b} {}_2F_1[c-a, c-b, c, z] . \quad (\text{B.2})$$

For  $a+b-c \neq m$ , where  $m \in \mathbf{Z}$

$$\begin{aligned} {}_2F_1[a, b, c, z] &= \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} {}_2F_1[a, b, a+b-c+1, 1-z] \\ &\quad (1-z)^{c-a-b} \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} {}_2F_1[c-a, c-b, c-a-b+1, 1-z] . \end{aligned} \quad (\text{B.3})$$

For  $c = a + b$  one obtains

$$\begin{aligned}
{}_2F_1[a, b, a + b, z] &= \frac{\Gamma(a + b)}{\Gamma(a)\Gamma(b)} \sum_{n=0}^{\infty} \frac{(a)_n(b)_n}{(n!)^2} \\
&\quad \times [2\psi(n + 1) - \psi(a + n) - \psi(b + n) - \ln(1 - z)] (1 - z)^n,
\end{aligned} \tag{B.4}$$

where  $\psi(z)$  is the Digamma function  $\psi(z) = \frac{d \ln \Gamma(z)}{dz}$  and  $(a)_n$  denotes Pochhammer's symbol  $(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)}$ .

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